



The Mathematics Department

7

Introducing

Angles

Name:

Contents

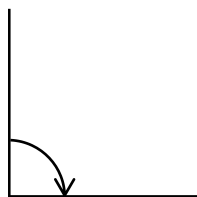
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Types of Angles



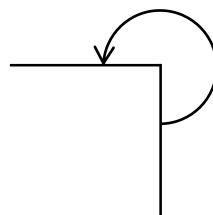
An angle measures **turn**. Here are some examples of different turns.



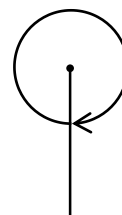
A quarter of a turn clockwise



Half a turn anticlockwise



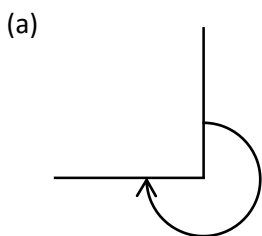
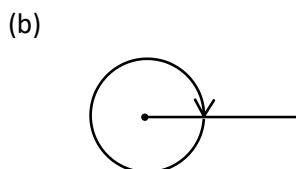
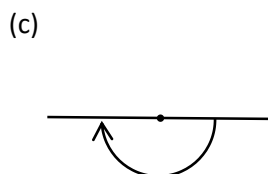
Three quarters of a turn anticlockwise



A full turn clockwise

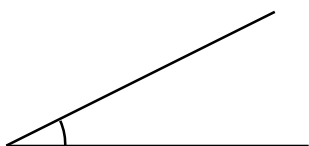
Exercise 1

What **type** of turns do you see below?



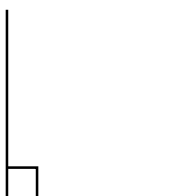
It is possible to measure an angle in degrees. There are **360°** in a full turn, so there are 180° in half a turn; 90° in a quarter of a turn, and so on.

We use the following names for the different types of angles.



Acute Angle

Any angle between 0° and 90°



Right Angle

An angle that is exactly 90°



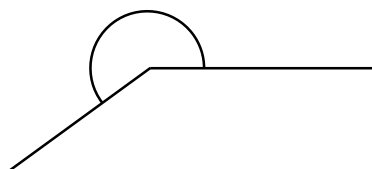
Obtuse Angle

Any angle between 90° and 180°



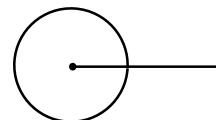
Straight Line

An angle that is exactly 180°



Reflex Angle

Any angle between 180° and 360°



Full Turn

An angle that is exactly 360°

Challenge!

Why do we use 360° for the size of a full turn, and not some other number?



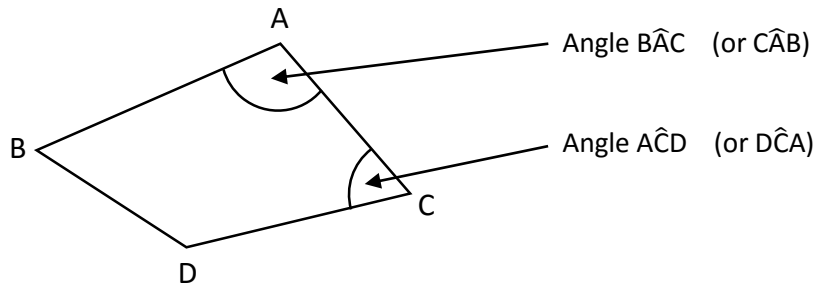
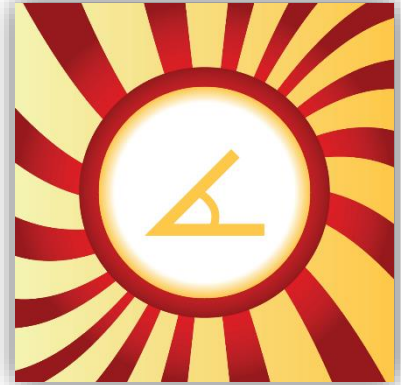
Exercise 2

What **type** of angles are the following?

- (a) 140° (b) 14° (c) 280° (d) 104° (e) 325° (f) 94° (g) 180°
(h) 54° (i) 205° (j) 90° (k) 270° (l) 2° (m) 360° (n) 103°

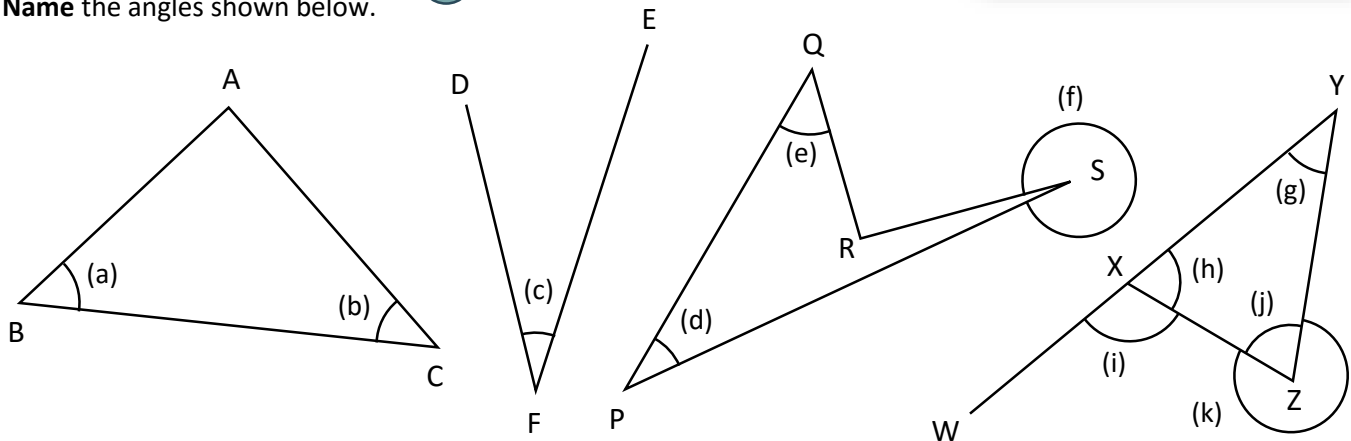
In order to **name** an angle, we label the vertices in a diagram with capital letters, before using **three letters** to identify a specific angle. (We need to add a **circumflex accent** to the letter in the middle.)

Example



Exercise 3

Name the angles shown below.



Exercise 4

What **type** of angles are the ones shown in Exercise 3?



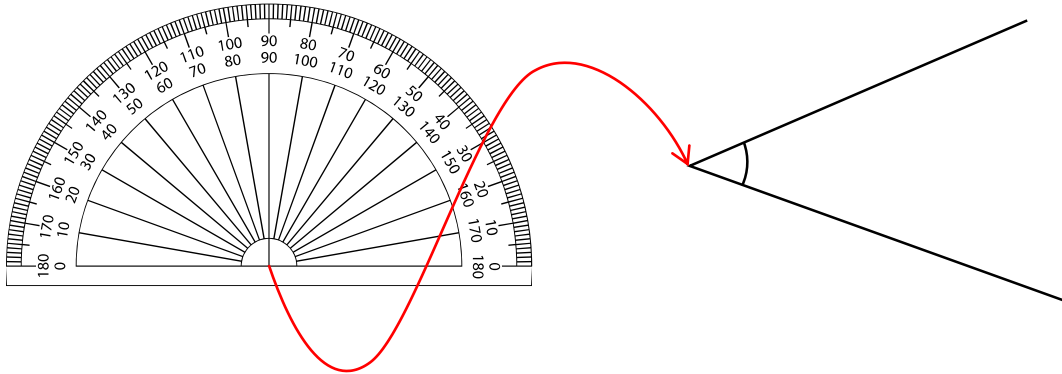
Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
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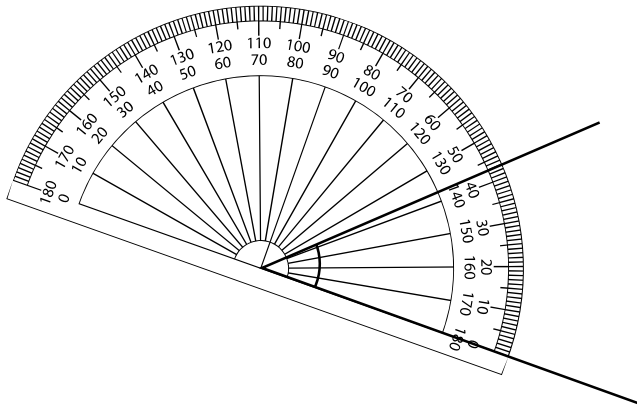
Measuring Angles

We use a **protractor** to measure angles. It is possible to obtain a protractor in the shape of half a circle (measures up to 180°) or in the shape of a full circle (measures up to 360°). To measure an angle:

(i) Put the **centre of the protractor** on the **vertex** (corner) of the angle.



(ii) **Turn** the protractor so that the angle 0° lies on one of the **lines** of the angle.

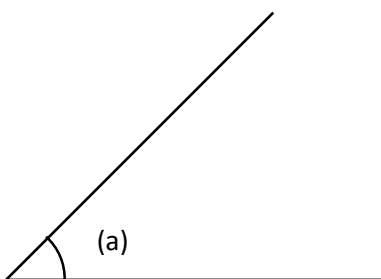
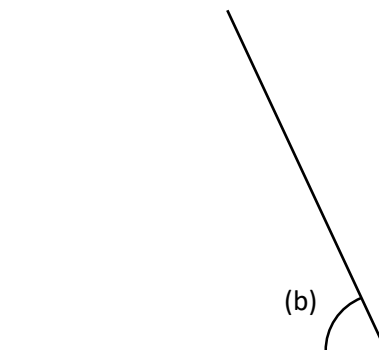
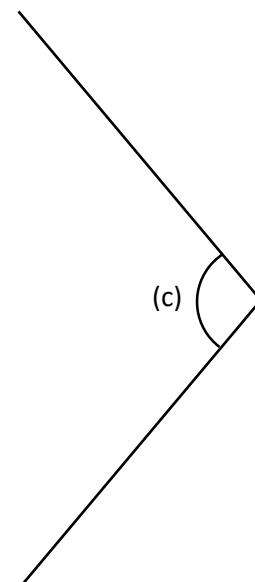


(iii) **Count up from 0°** in order to measure the angle. (It is important to do this as there are two sets of numbers on a protractor.) What angle is shown by the protractor above?

Exercise 5

5

Estimate the size of each of the following angles in degrees. Write your estimate down before **measuring** the angle using a protractor. How accurate were you in estimating the size of the angles?

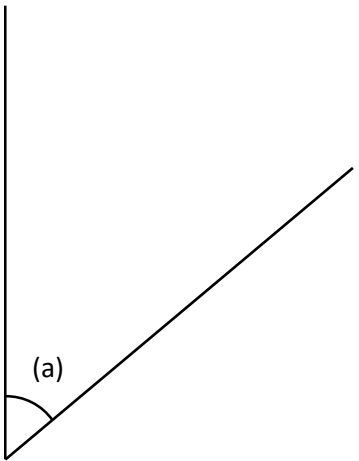
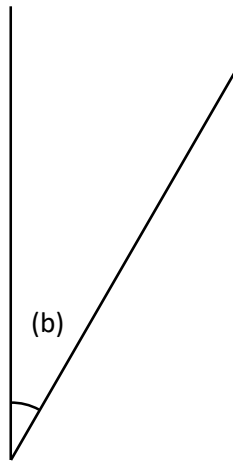
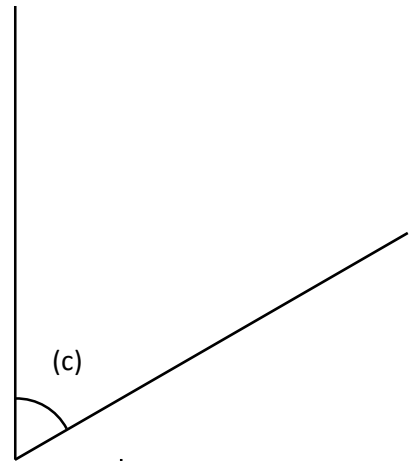
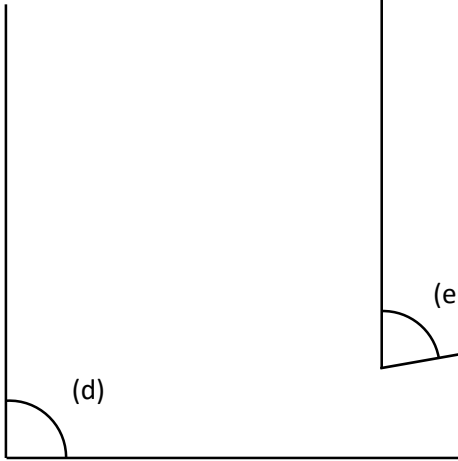
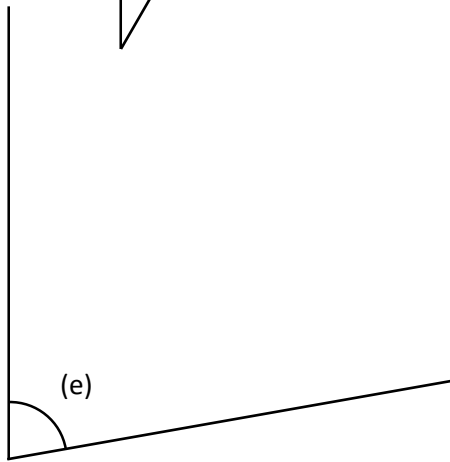
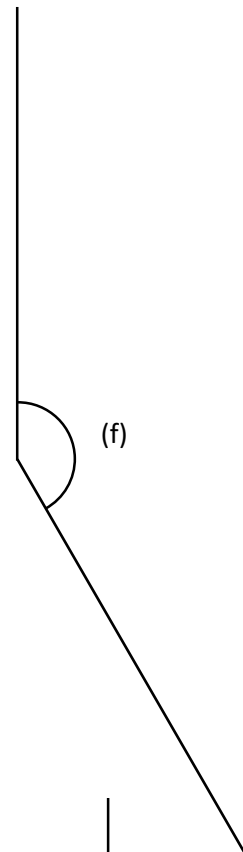
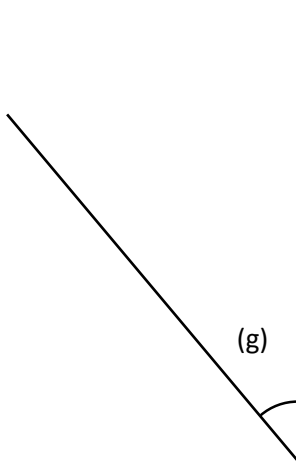
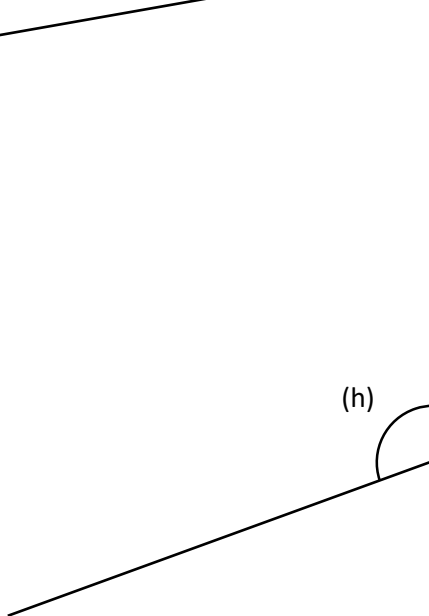
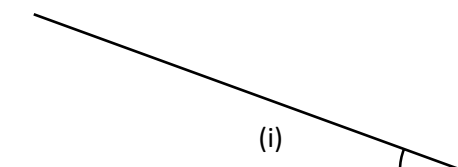
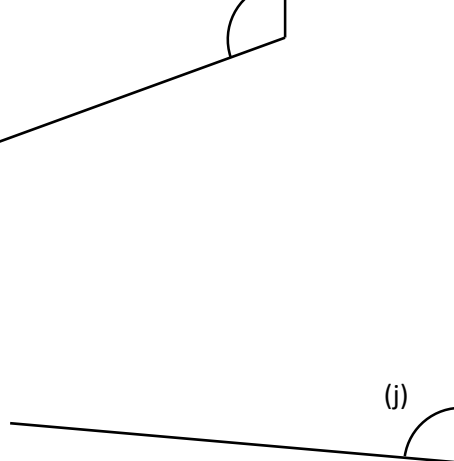


Exercise 6

5

Skill

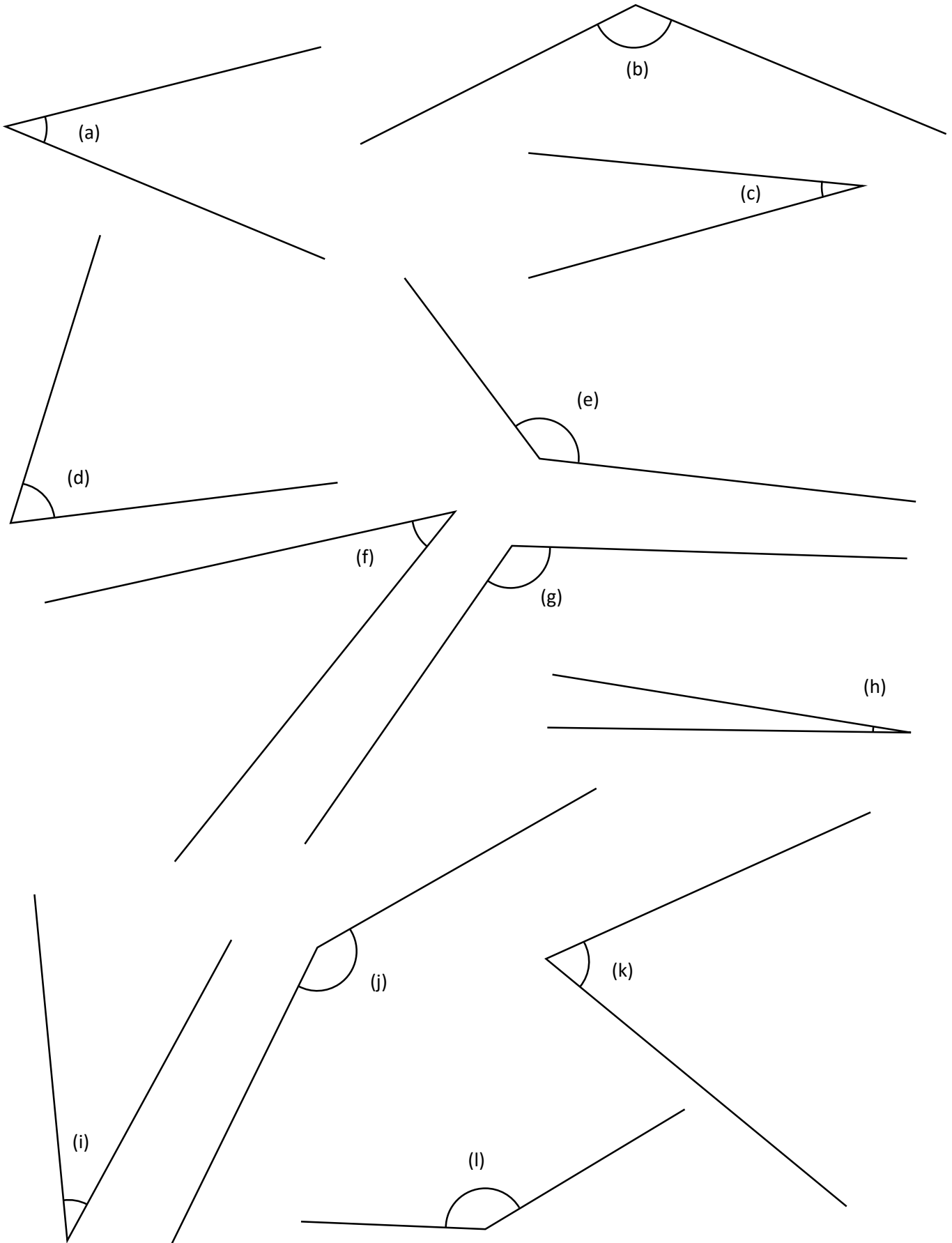
Measure the following angles.



Exercise 7

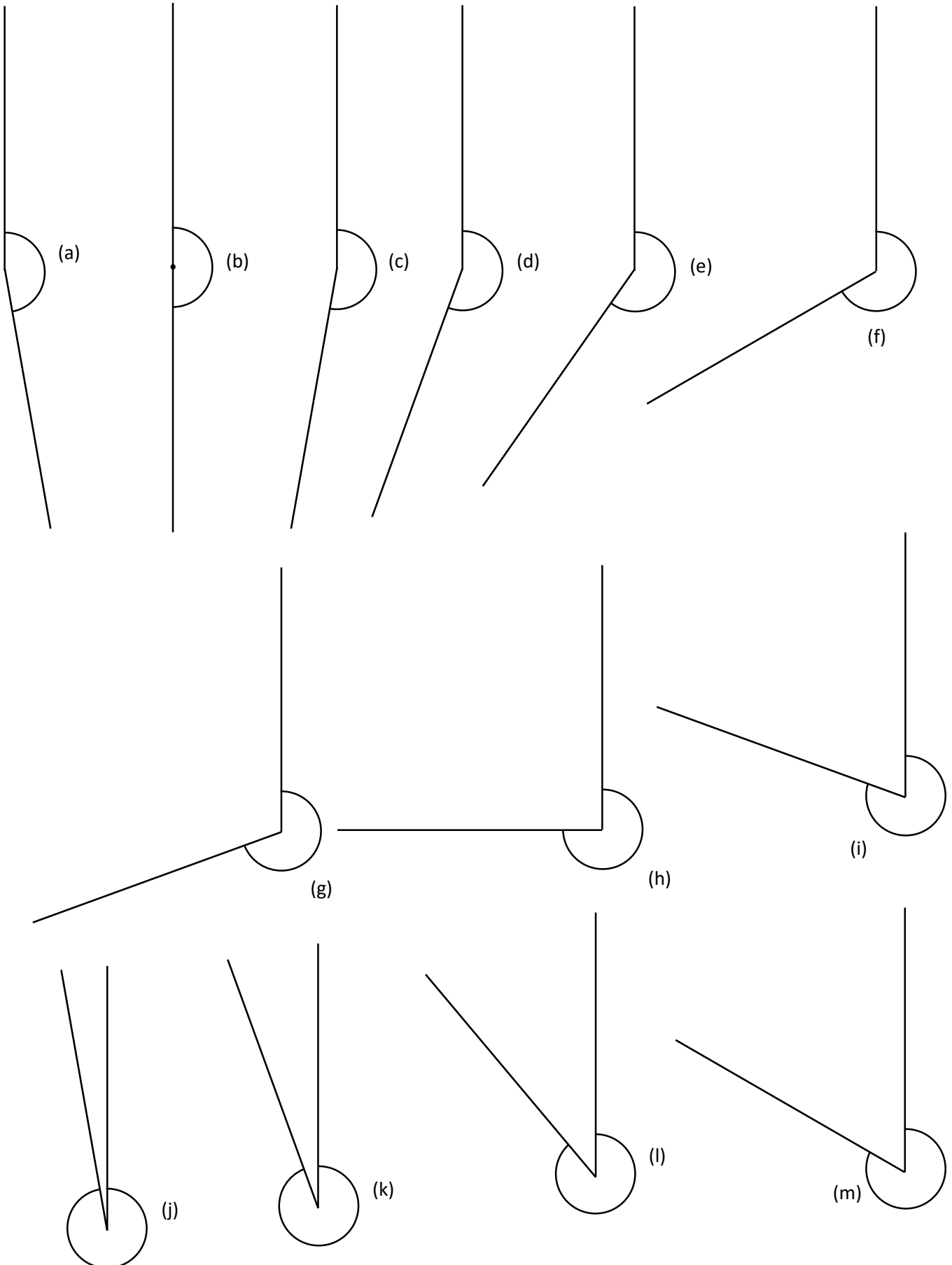
5

Measure the following angles and state what **type** of angles they are.



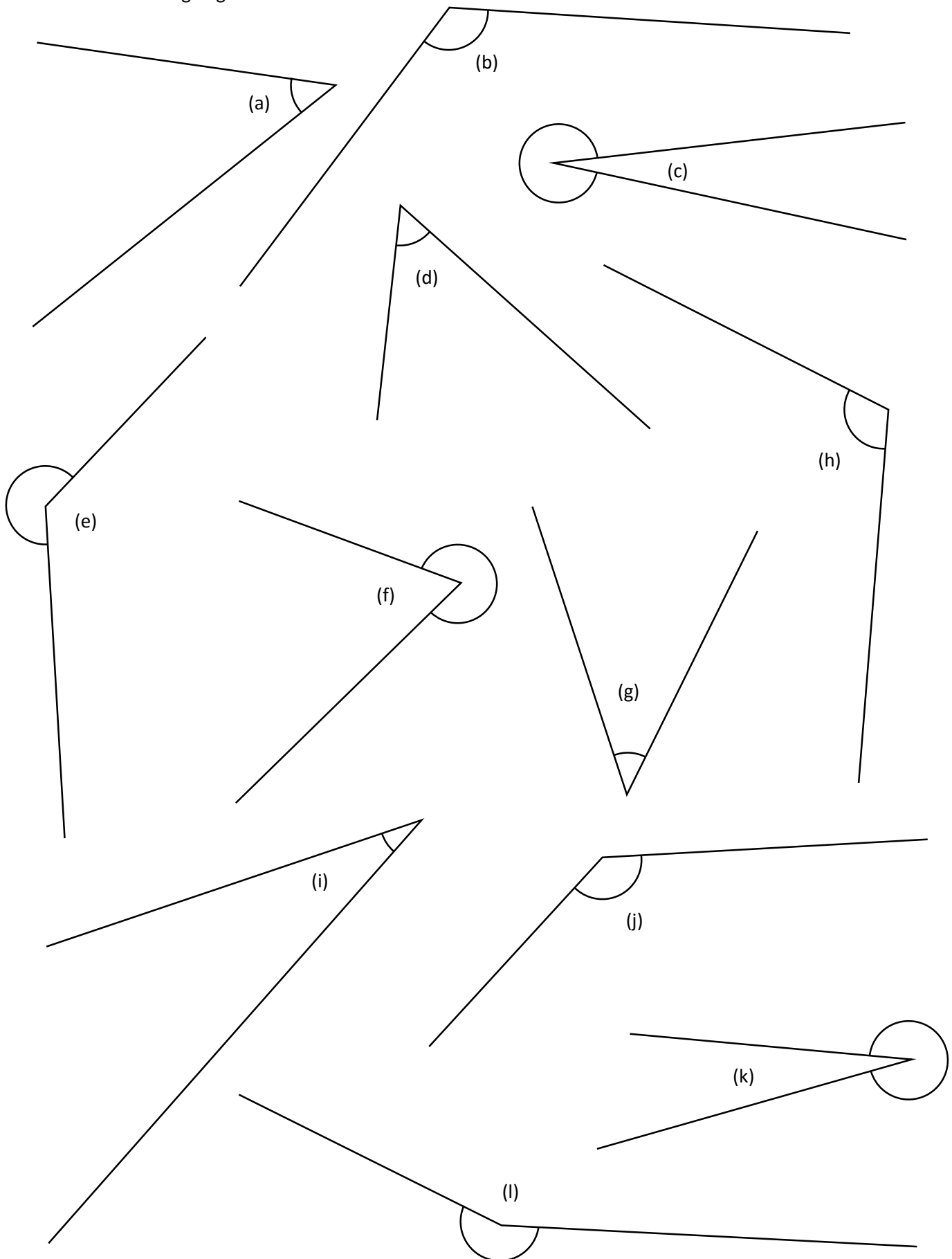
Exercise 8

5

Measure the following angles.

Exercise 9

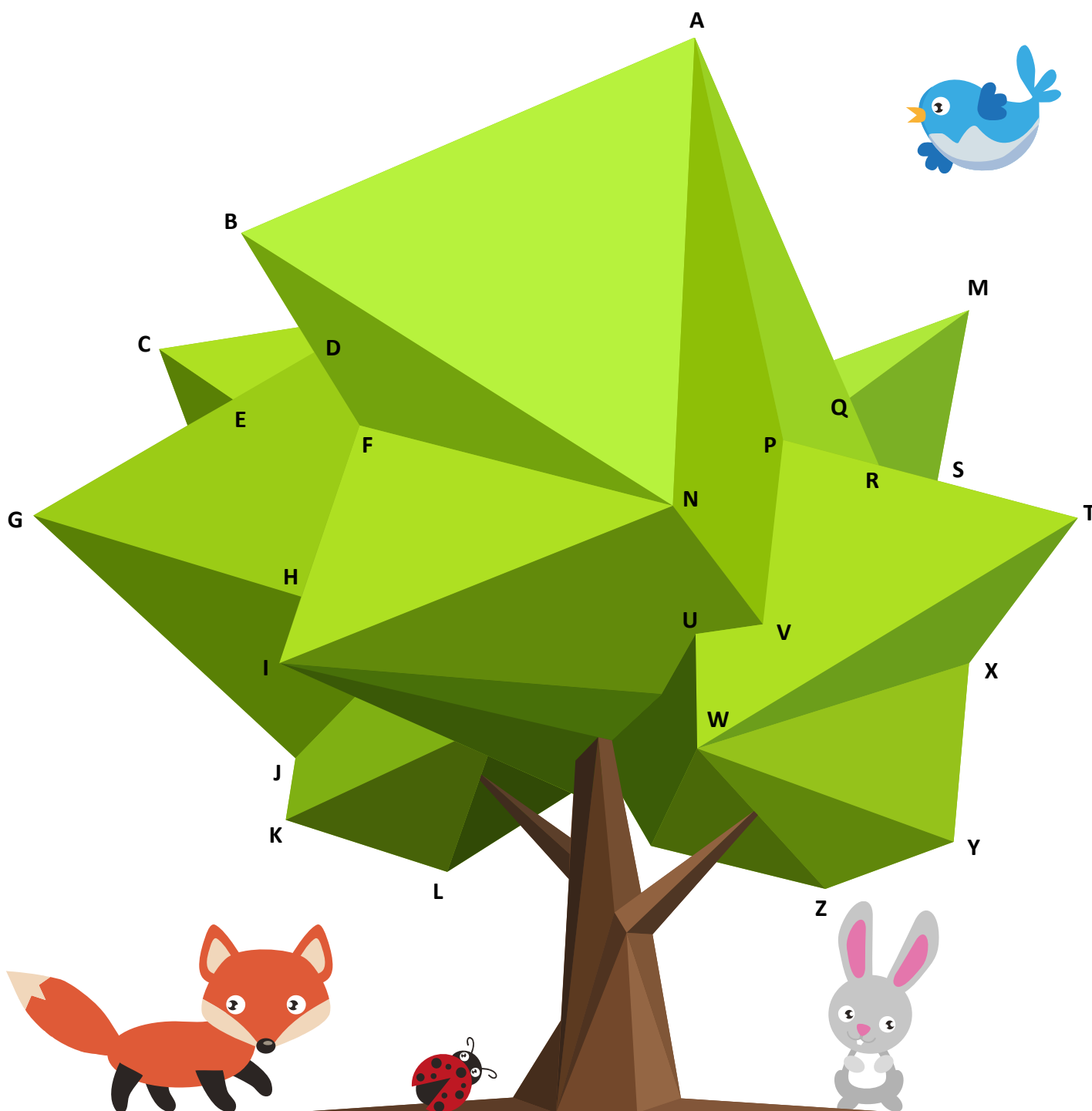
5

Measure the following angles.

Exercise 10**Measure** the following angles.

5

Applying

(a) $\widehat{BAN}$ (b) $\widehat{ABN}$ (c) $\widehat{ANB}$ (d) $\widehat{BFH}$ (e) $\widehat{HGE}$ (f) $\widehat{NIF}$ (g) $\widehat{PTW}$ (h) Reflex angle $\widehat{PTW}$ (i) $\widehat{QMS}$ (j) $\widehat{TXW}$ (k) $\widehat{WUV}$ (l) $\widehat{VNA}$ (m) Reflex angle $\widehat{HGJ}$ (n) $\widehat{LKJ}$ (o) Reflex angle $\widehat{ZYX}$ 

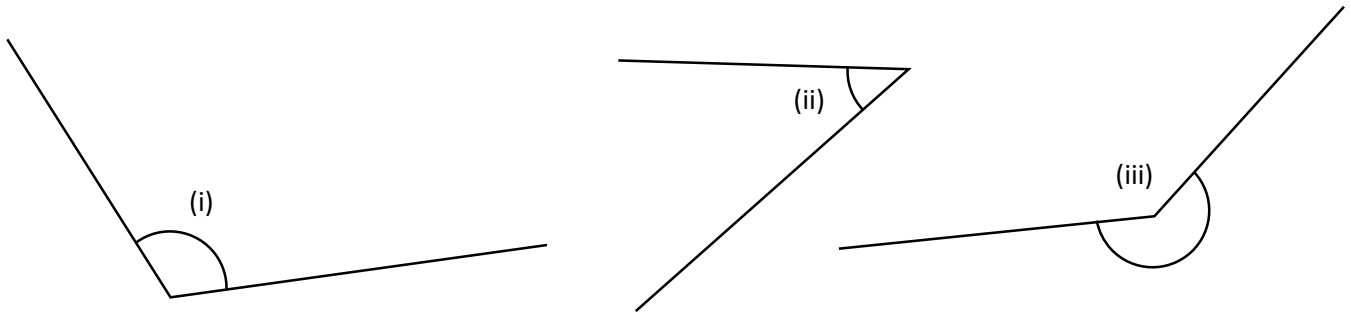
Exercise 11 (Revision)

(a) **Estimate** the size of each of the following angles in degrees.

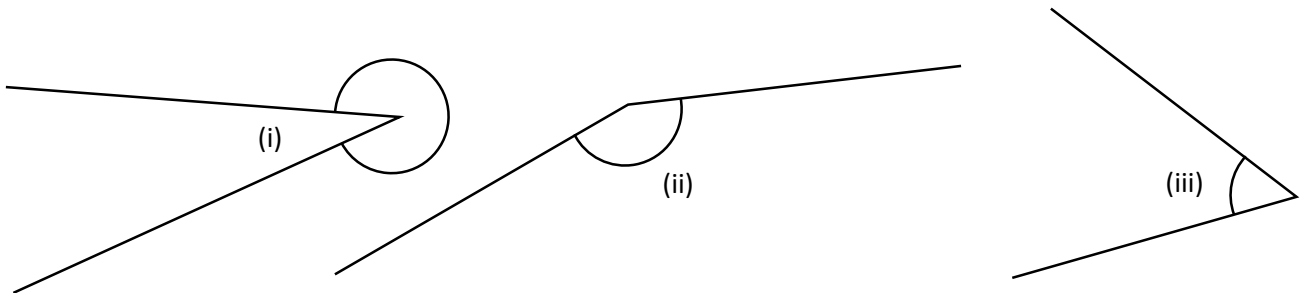
Write down your estimate before **measuring** the size of the angle using a protractor.

5

Skill



(b) **Measure** the following angles and state what **type** of angles they are.

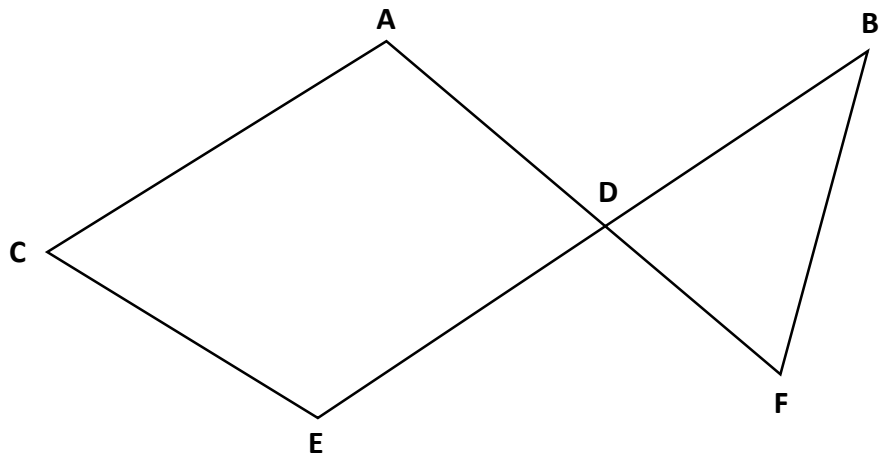


(c) **Measure** the following angles.

(i) $\widehat{ADE}$

(ii) $\widehat{DBF}$

(iii) Reflex angle $\widehat{ACE}$



Evaluation

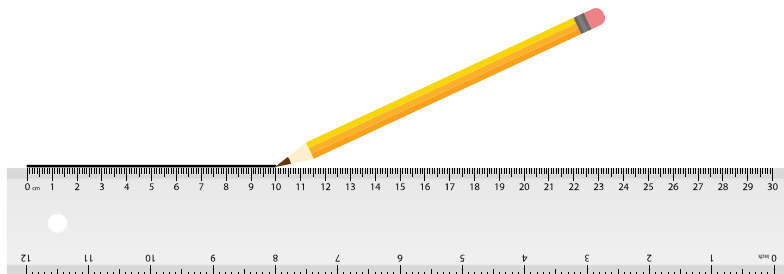
Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Drawing Angles

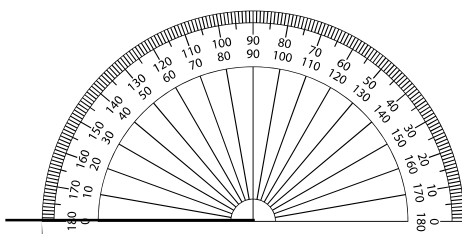
It is possible to draw any angle using a protractor, and it is possible to draw some special angles using a compass.

Drawing Angles Using a Protractor

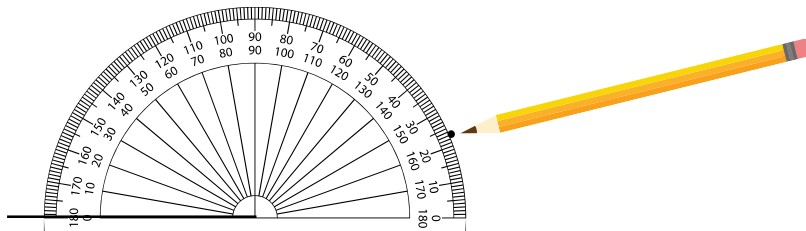
- (1) Draw a straight line using a ruler. (Allow plenty of space around the line in order to draw the angle.)



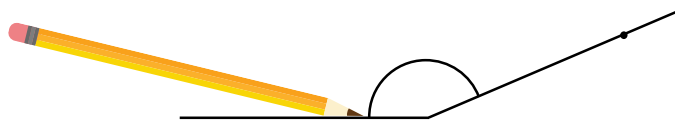
- (2) Place the centre of the protractor on one end of the line. Turn the protractor so that the angle 0° lies on the line.



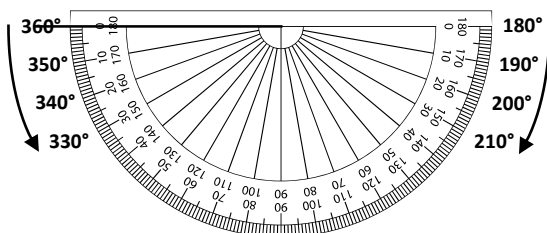
- (3) Count up from 0° to the angle you are attempting to draw. Mark the angle (e.g. 157°) using a pencil.



- (4) Draw a straight line through the dot, starting from where the centre of your protractor was. Draw the arc of the angle.



Note: If you are drawing a **reflex** angle, hold your protractor as shown below during step (2). Then, during step (3), either count up from 180° on the right side of the protractor, or count down from 360° on the left side of the protractor.



Exercise 12

Draw the following angles using a protractor.

- | | | | | | | |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| (a) 13° | (b) 40° | (c) 78° | (d) 93° | (e) 107° | (f) 138° | (g) 164° |
| (h) 193° | (i) 216° | (j) 254° | (k) 265° | (l) 284° | (m) 303° | (n) 341° |

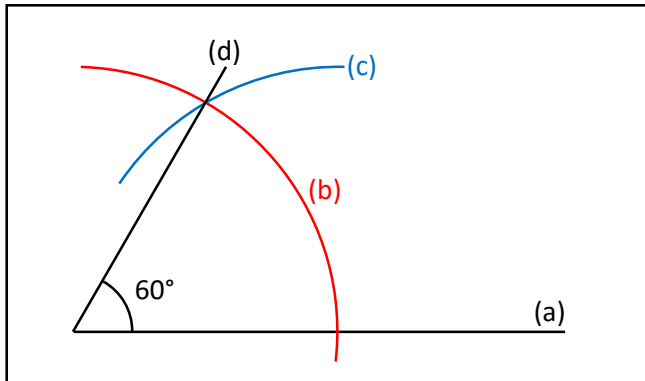
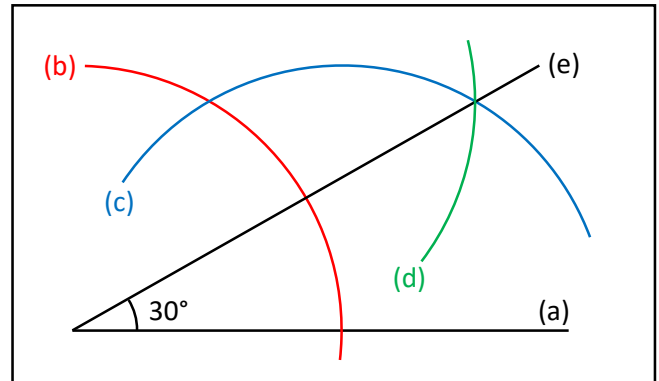
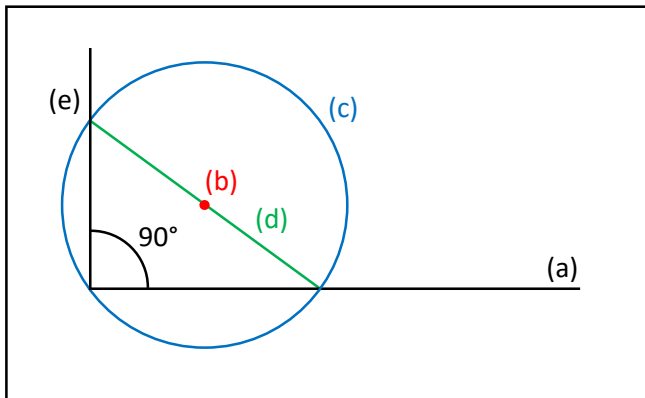
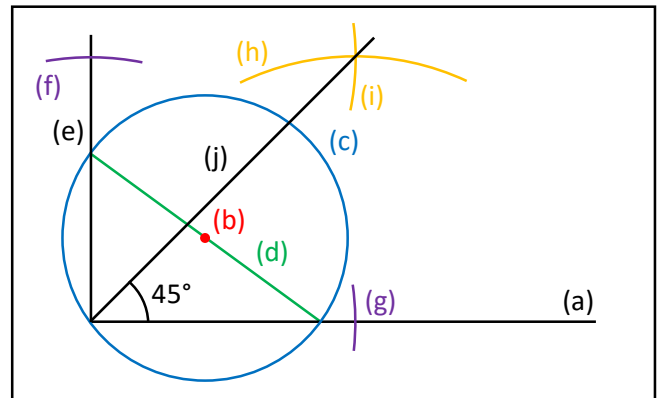
5

Skill

Drawing Angles Using a Compass

It is possible to use a compass and a ruler only to draw the angles 30° , 45° , 60° and 90° .

Do not erase the construction arcs after completing an angle!

**Exercise 13**

Draw the angles 60° , 30° , 90° and 45° using only a compass and a ruler.

5

Exercise 14 (Revision)

(a) Draw the following angles using a protractor.

(i) 25°

(ii) 74°

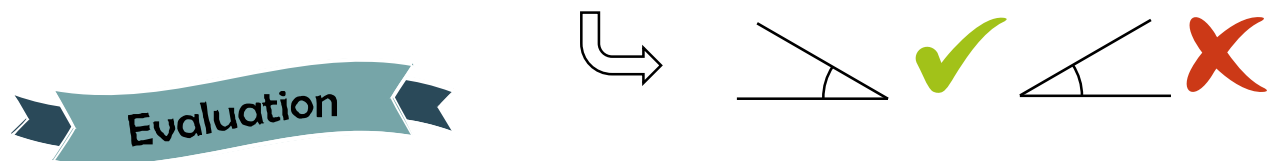
(iii) 172°

(iv) 208°

(v) 270°

(vi) 317°

(b) Repeat Exercise 13, but draw your angles so that the mouth of the angle points to the left, not to the right.



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...

My Level ☐ My Target ☐

Calculating Angles

With enough information, it is possible to find the size of some angles without using a protractor, by **calculation**.

Angles Around a Point

From the information on page 3, we know that a complete turn is 360° . It follows that the **angles around any point** must sum to 360° .

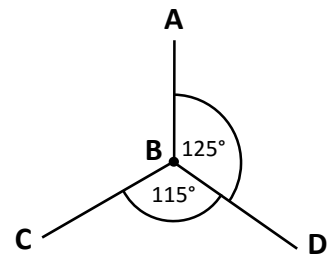
Example

In the diagram on the right, it is possible to find the size of the angle $\hat{A}BC$ without using a protractor, as the sum of $\hat{A}BC$, $\hat{C}BD$ and $\hat{A}BD$ must be 360° .

$$125^\circ + 115^\circ = 240^\circ$$

$$\hat{A}BC = 360^\circ - 240^\circ$$

$$\hat{A}BC = 120^\circ.$$

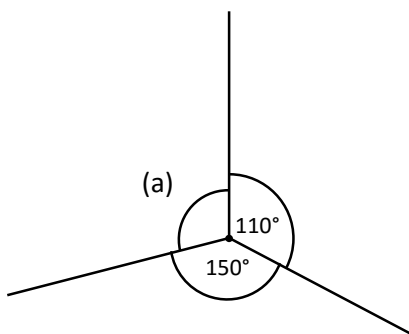
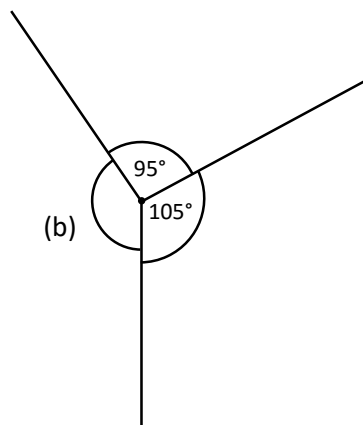
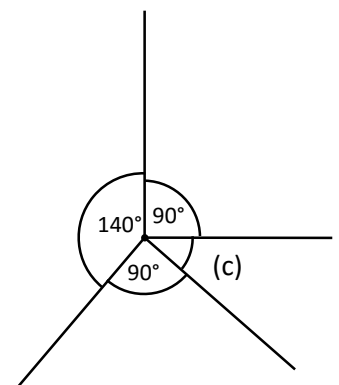
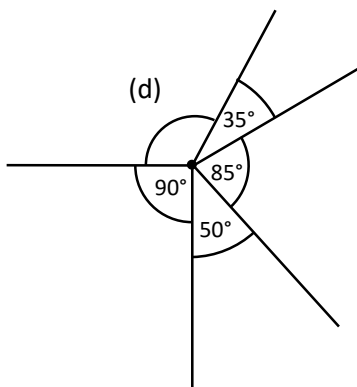
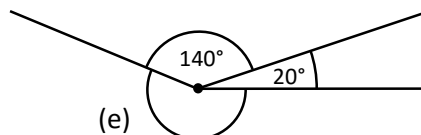
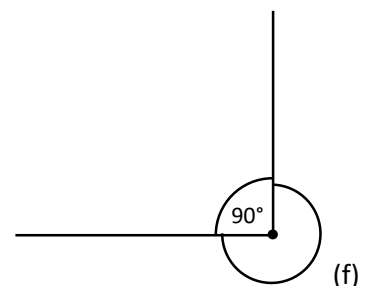
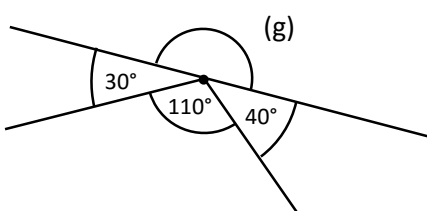
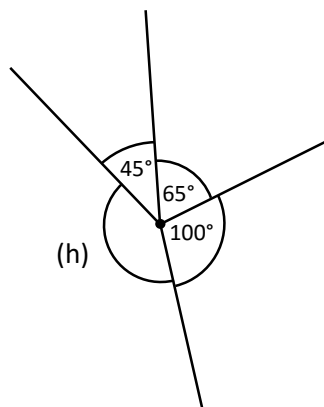
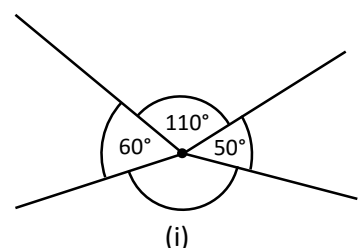


6

Skill

Exercise 15

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)



Angles on a Straight Line

The angle for half a turn is 180° , so the angles on any **straight line** must sum to 180° .

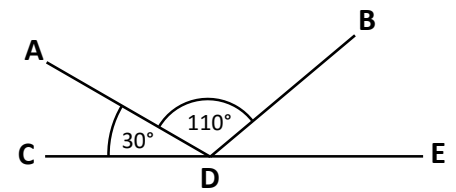
Example

In the diagram on the right, it is possible to find the size of the angle $\widehat{BDE}$ without using a protractor, as the sum of $\widehat{ADC}$, $\widehat{ADB}$ and $\widehat{BDE}$ must be 180° .

$$30^\circ + 110^\circ = 140^\circ$$

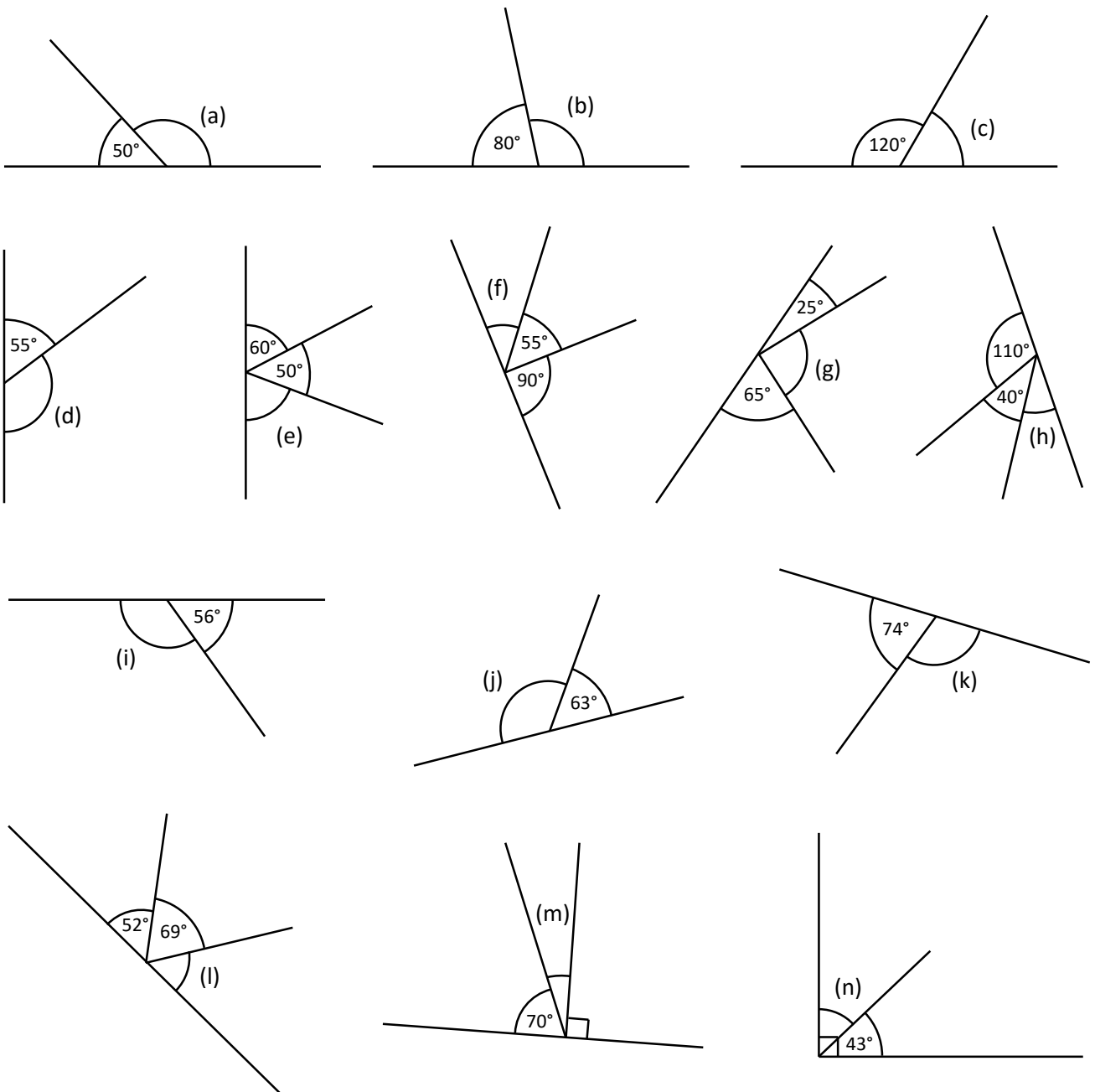
$$\widehat{BDE} = 180^\circ - 140^\circ$$

$$\widehat{BDE} = 40^\circ.$$



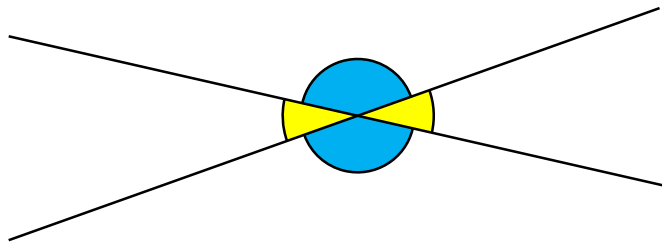
Exercise 16

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)



Vertically Opposite Angles

Every time that two lines cross, four angles are formed around the vertex. The angles that are opposite each other (not next to each other) are **equal**. We call these angles **vertically opposite angles**.

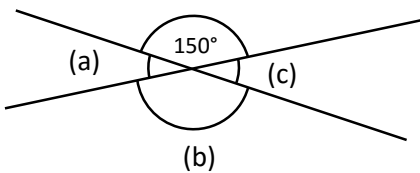
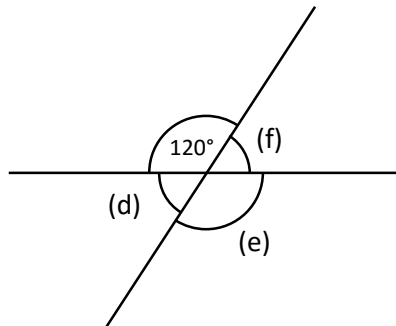
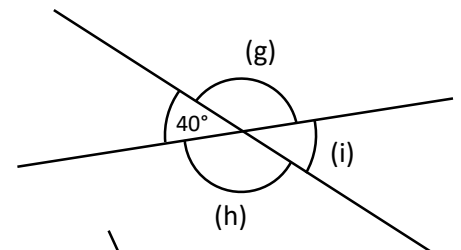
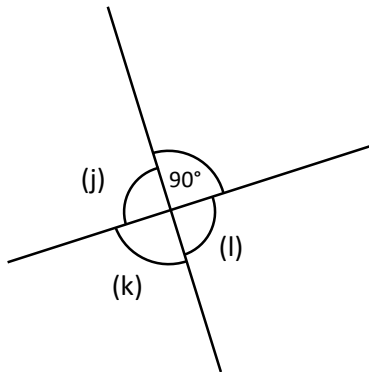
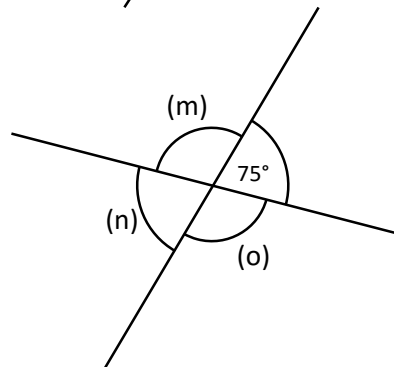
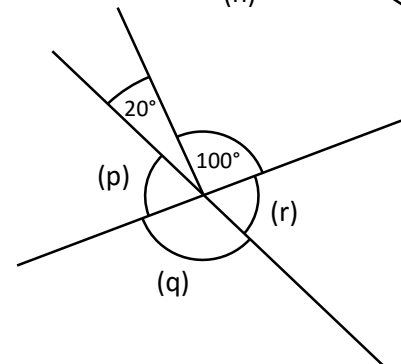
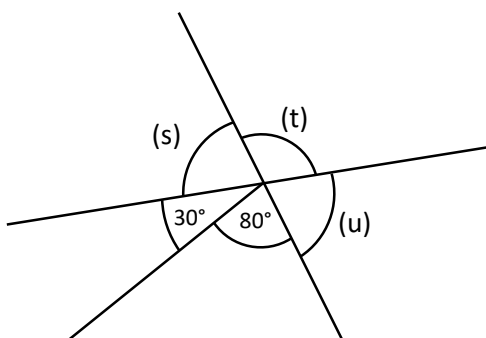
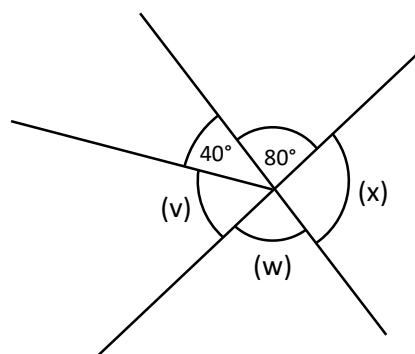
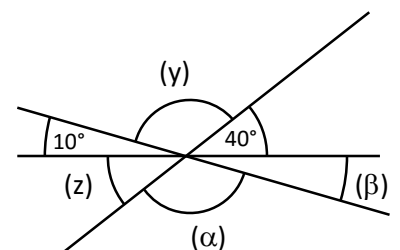


Exercise 17

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)

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Skill



Challenge!

The width of a football goal is 8 yards. The shortest distance from the goal to the penalty spot is 12 yards. Construct a scale drawing in order to find the angle shown in the picture, which is the angle to score a penalty.



Applying

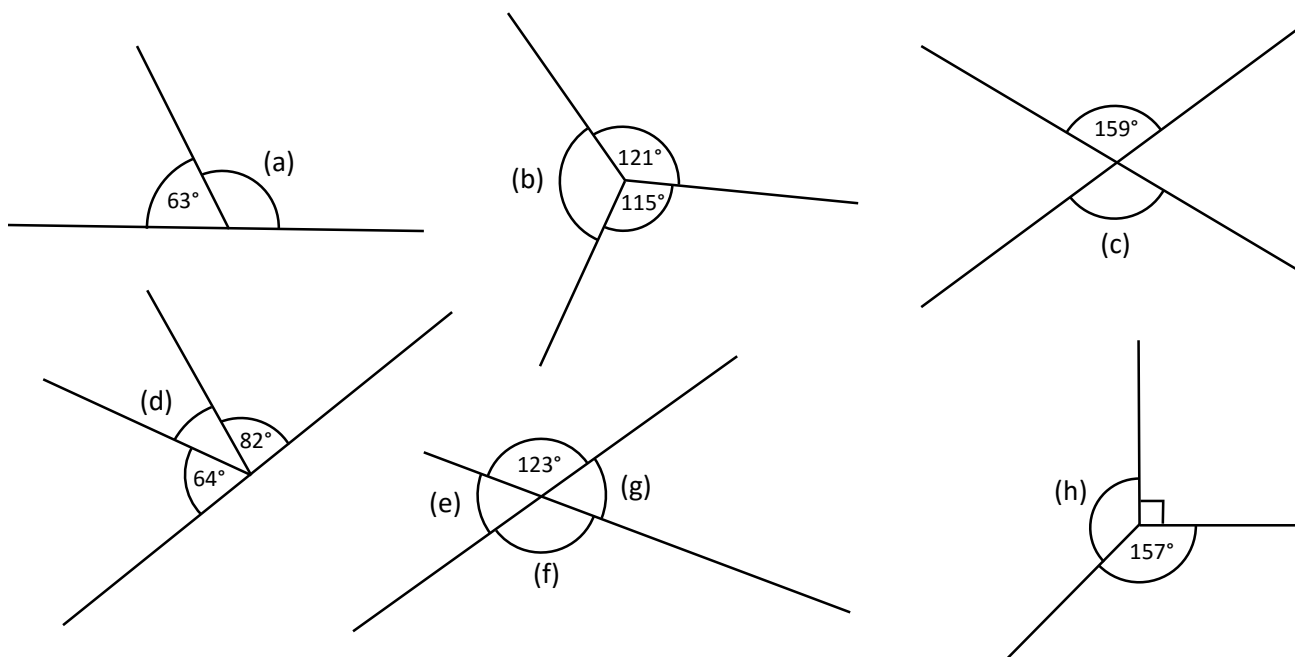


Exercise 18

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)

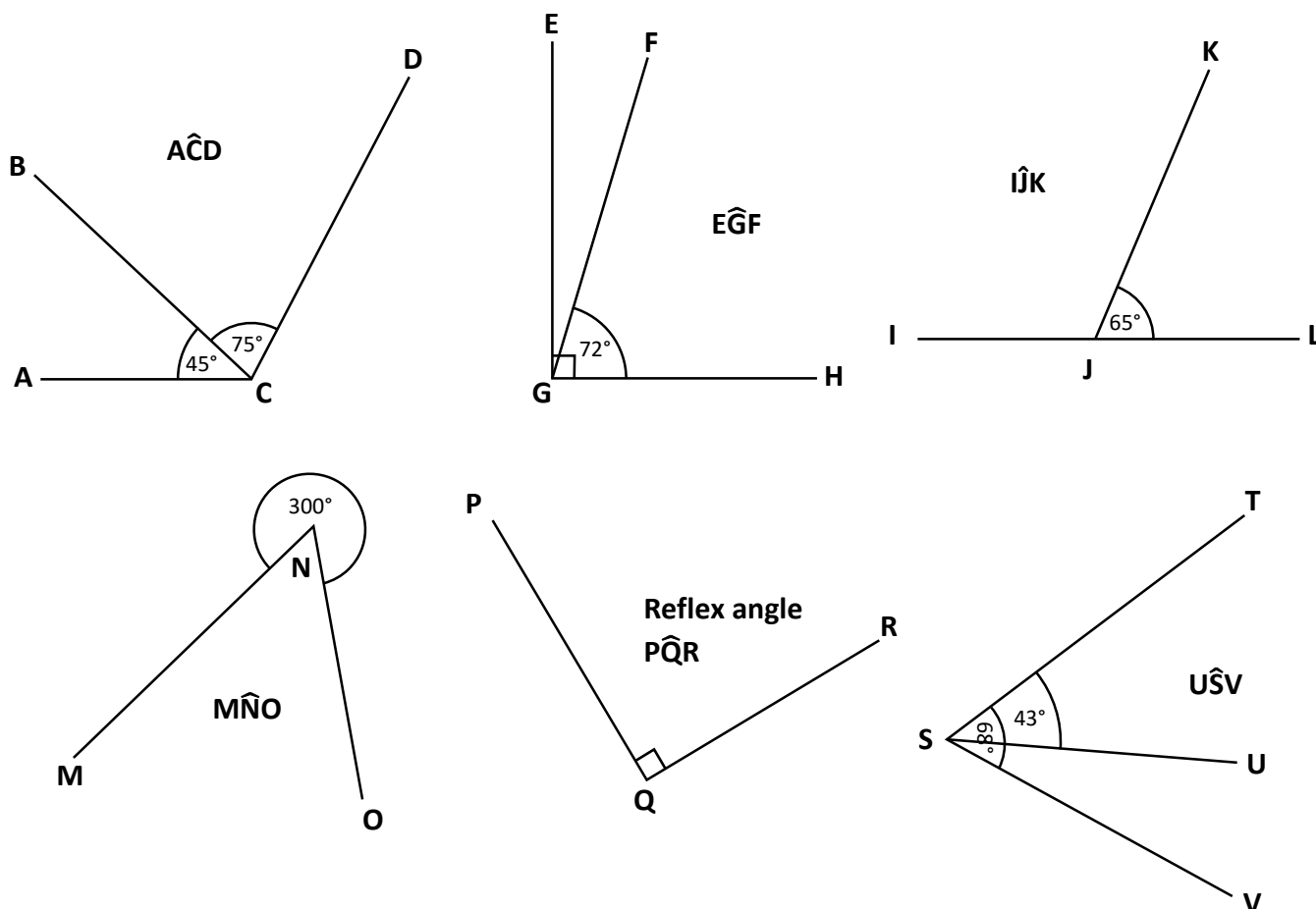
6

Skill

**Exercise 19**

Calculate the **size** of each of the named angles. (The diagrams are not drawn to scale.)

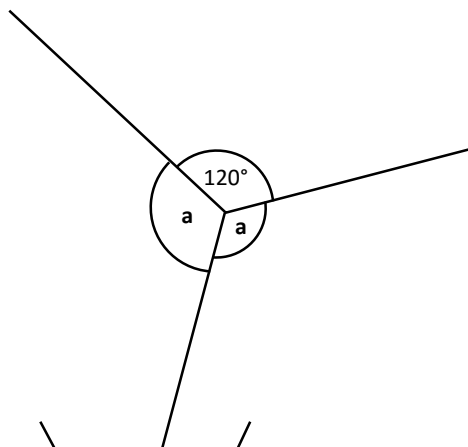
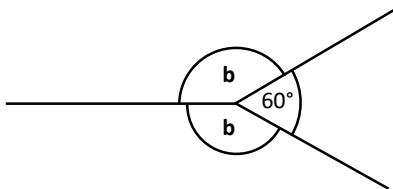
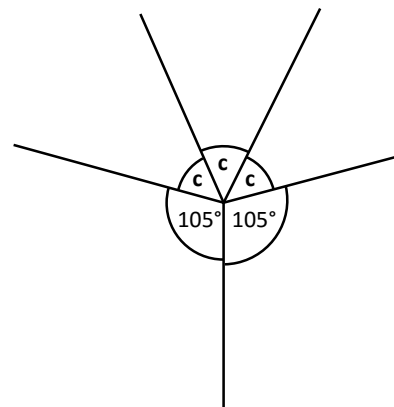
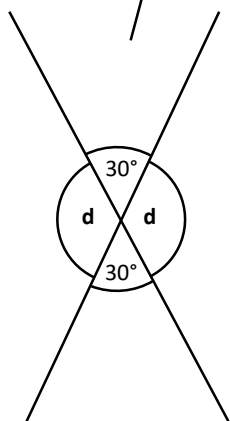
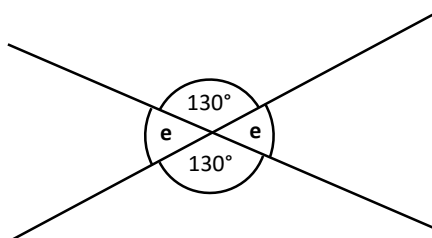
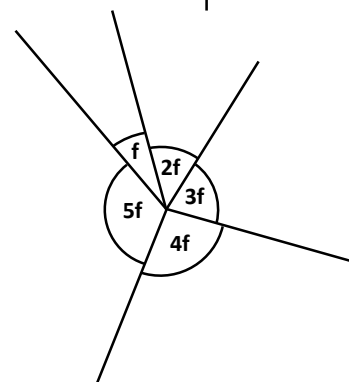
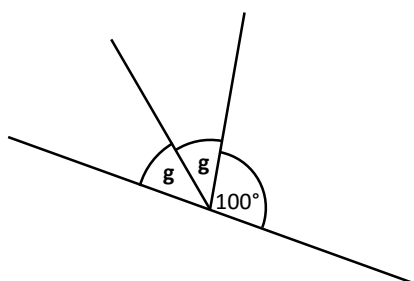
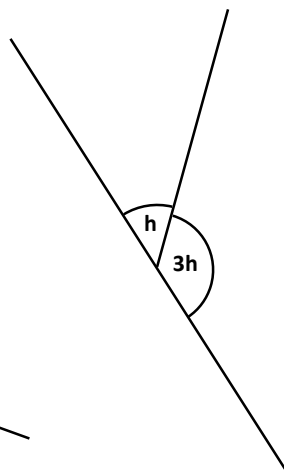
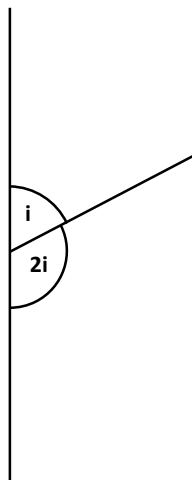
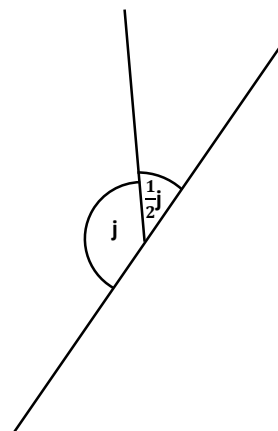
6



Can you find the following angles? (The diagrams are not drawn to scale.)

6

Extension



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			<div> My Level ☐ My Target ☐ </div>

Angles in a Triangle

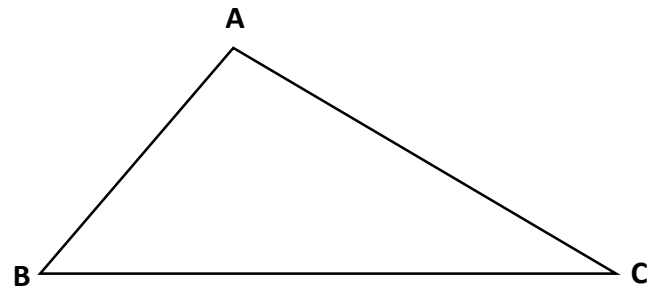
The angles inside a shape are called **interior angles**.

Use your protractor to measure the three interior angles of the triangle shown on the right.

$$\hat{A}BC = \underline{\hspace{2cm}}^\circ$$

$$\hat{B}AC = \underline{\hspace{2cm}}^\circ$$

$$\hat{A}CB = \underline{\hspace{2cm}}^\circ$$



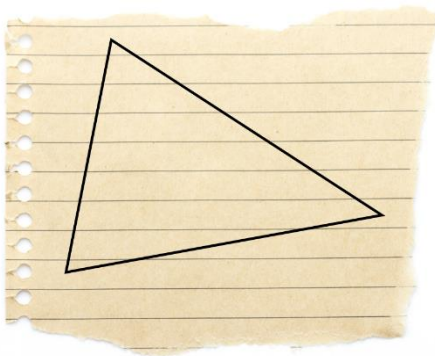
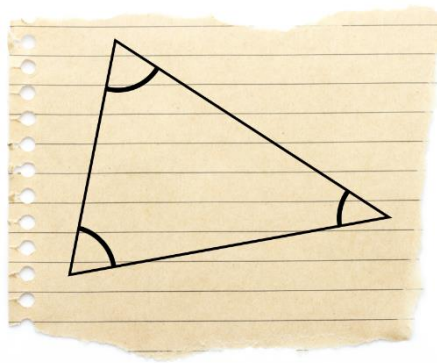
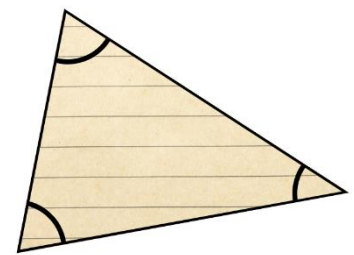
What is the total of your answers?

$$\text{Total} = \underline{\hspace{2cm}}^\circ$$

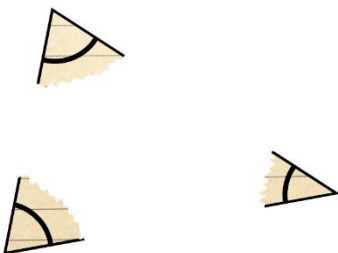
Proving the Sum of the Interior Angles of a Triangle

Follow the steps below to show that the sum of the interior angles of a triangle is always the same.

1. Draw any triangle on a piece of scrap paper. The triangle does **not** need to look like the triangle below.
2. Mark the interior angles of your triangle with thick arcs.
3. Cut your triangle out using a pair of scissors.



4. Tear off the corners of your triangle.



5. Arrange the corners to form a straight line.



6. Glue your corners (in a straight line) in the box below.



7. Complete the following sentence:

The sum of the interior angles of a triangle is always $\underline{\hspace{2cm}}^\circ$



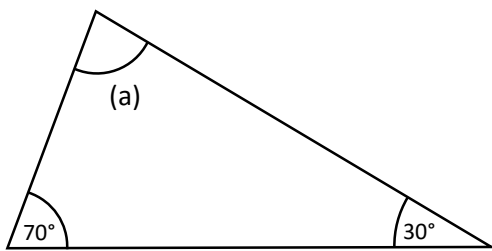
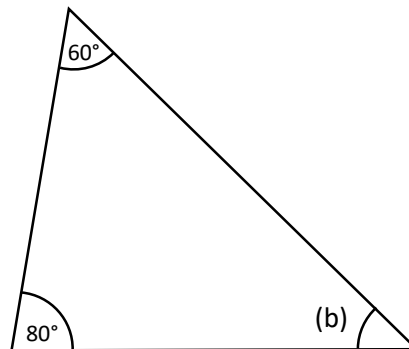
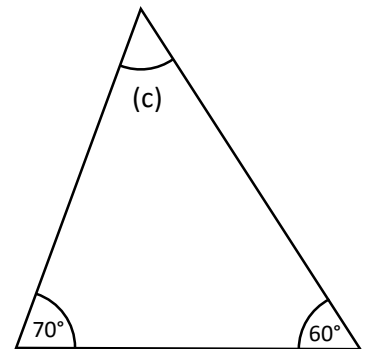
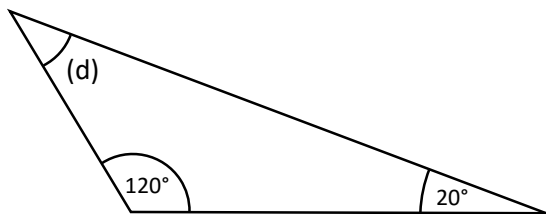
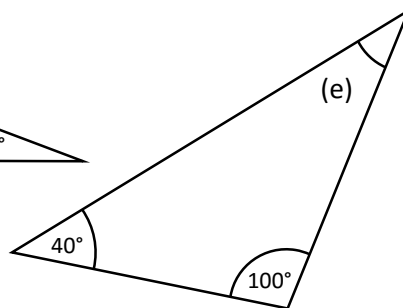
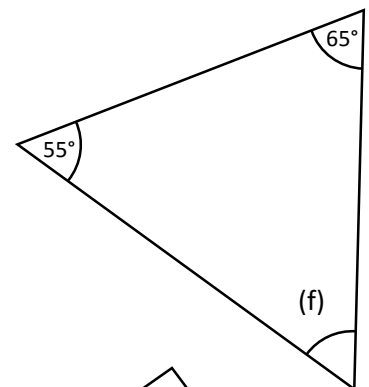
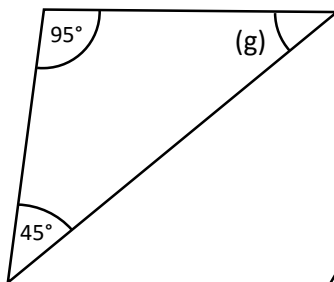
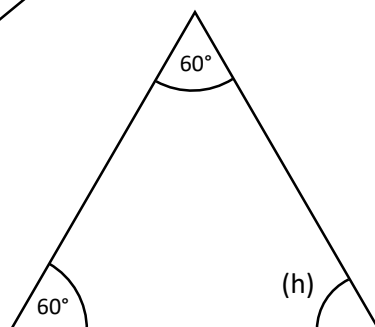
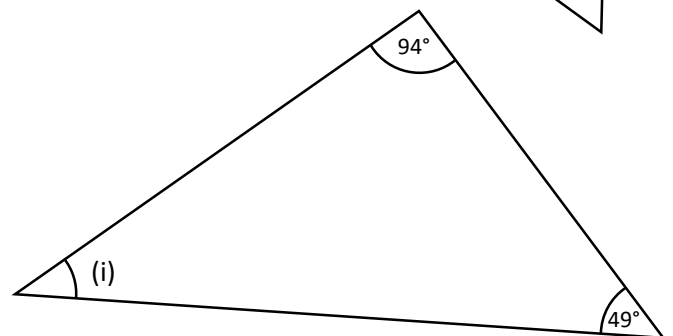
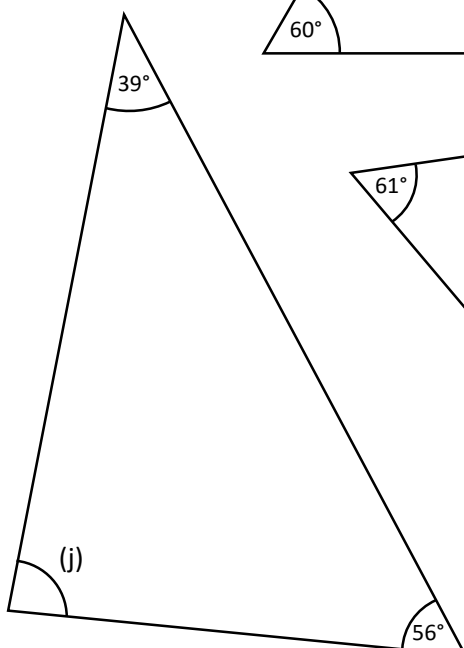
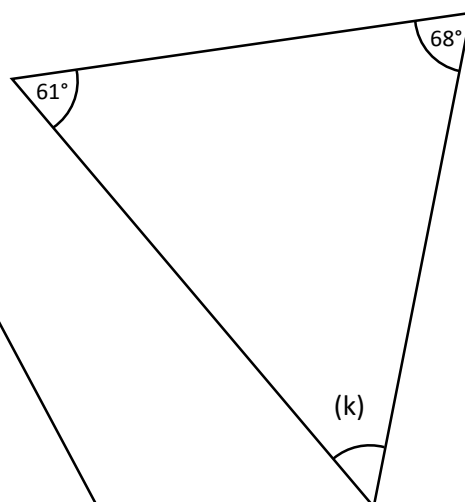
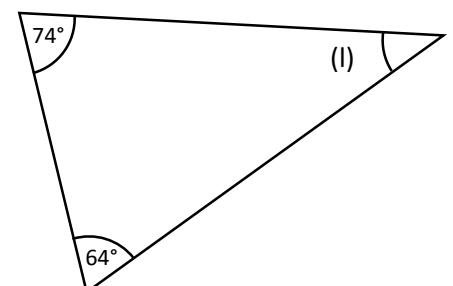
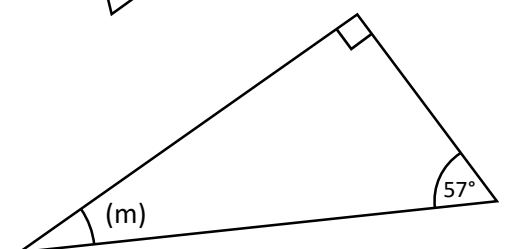
Exercise 20

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)



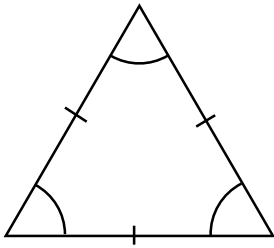
6

Skill

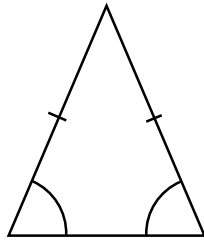


Types of Triangle

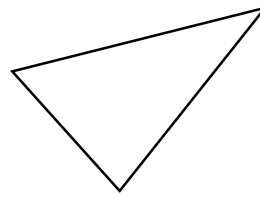
An **equilateral** triangle has three equal sides and three equal angles.



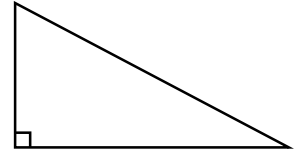
An **isosceles** triangle has two equal sides and two equal angles.



A **scalene** triangle does not have any equal sides or any equal angles.



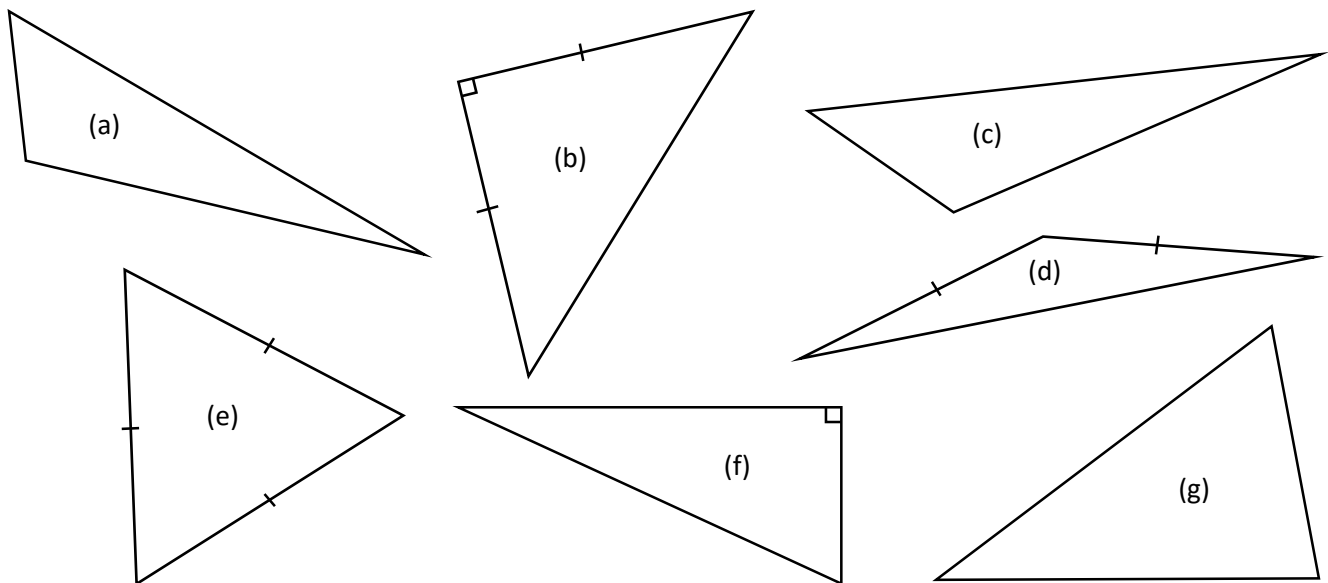
A **right-angled** triangle has one angle that is a right angle, or 90° .

**Exercise 21**

What **type** of triangles are shown below?

6

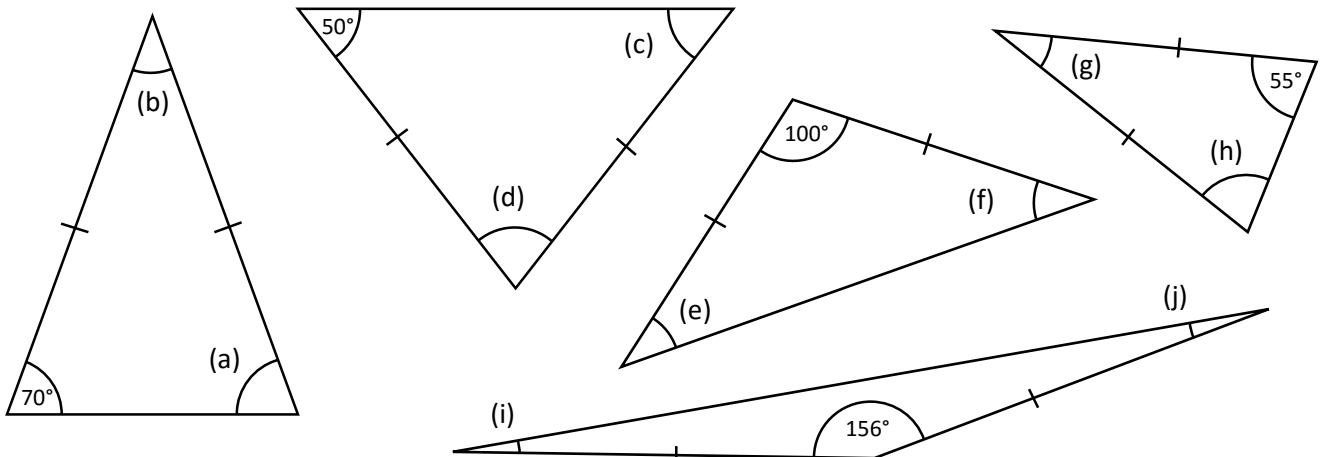
Skill

**Exercise 22**

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)



6



Exercise 23

(a) Dewi draws a triangle with angles 50° , 80° and 50° .

What type of triangle is this?

(b) Explain whether or not it is possible to draw a triangle with two obtuse angles.

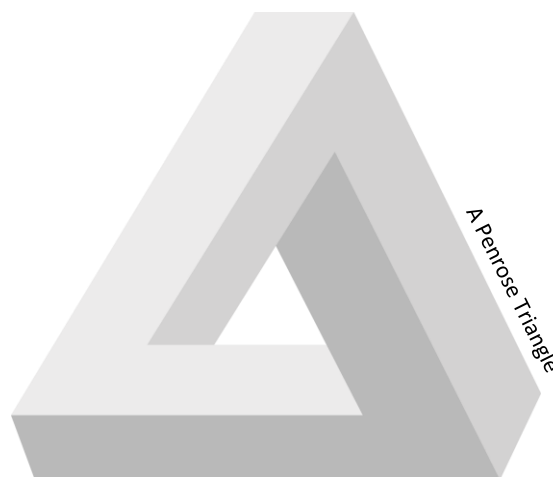
(c) Llinos draws an isosceles triangle that is also a right-angled triangle.

What are the size of the angles in Llinos' triangle?

(d) Explain whether or not it is possible to draw an equilateral triangle that is also a right-angled triangle.

(e) Tomos draws a triangle and measures its angles as 95° , 95° and 170° .

Erin measures the angles of the same triangle and gets 85° , 85° and 10° . Explain who has measured the angles correctly. What mistake has the other person made?

**Investigation**

Draw triangles on the dotted paper below (each corner of a triangle must be placed on a dot).

Identify the type of triangle underneath the dots. How many different triangles are possible?

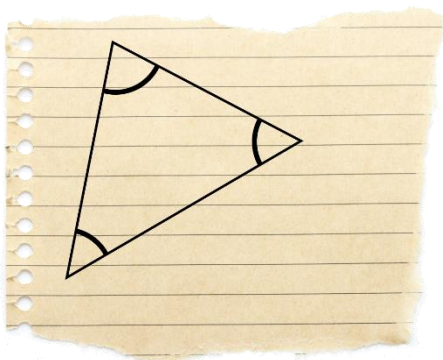
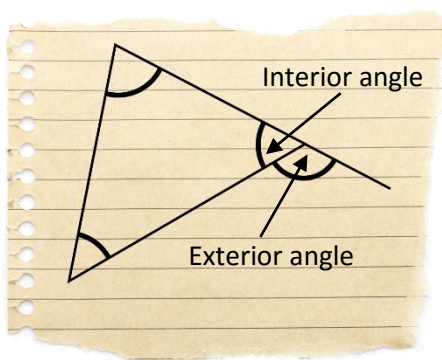
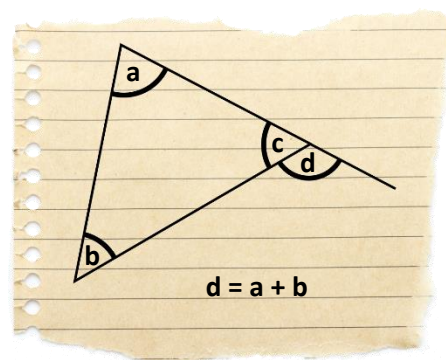
<p>Isosceles; Right-Angled</p>			

The Exterior Angle of a Triangle

Consider any triangle.

By extending one of the sides of the triangle, we create an **exterior angle**.

The size of the exterior angle is the sum of the **two opposite interior angles**.

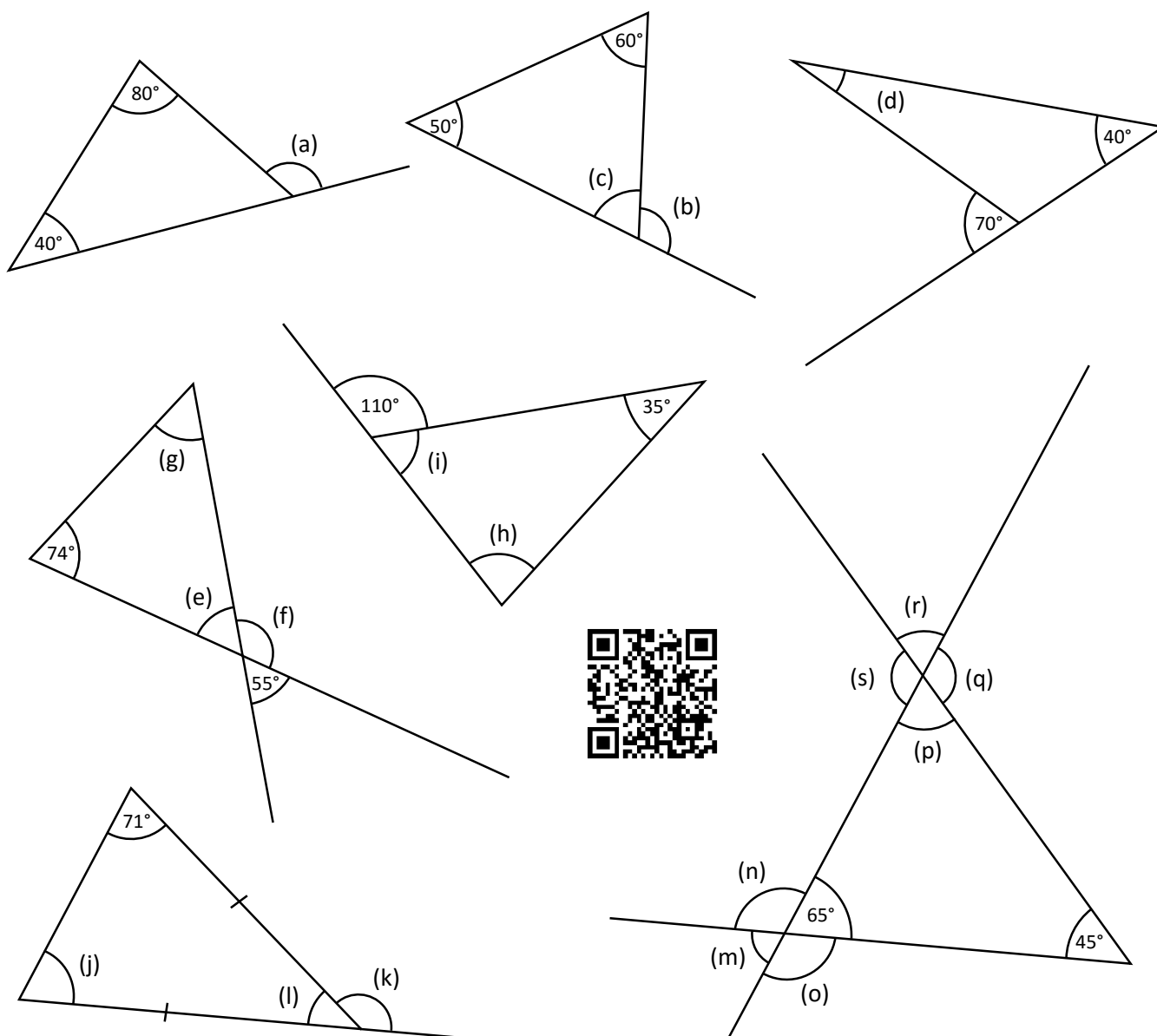


Exercise 24

Calculate the **size** of the marked angles. (The diagrams are not drawn to scale.)

6

Skill



Constructing Triangles

Using a ruler, a protractor and a compass, it is possible to draw any triangle if we know one of the following.

- The length of **one side** of the triangle and the size of **two of the angles**.
- The length of **two sides** of the triangle and the size of the **angle joining** these two sides.
- The length of **all three sides** of the triangle.

Exercise 25

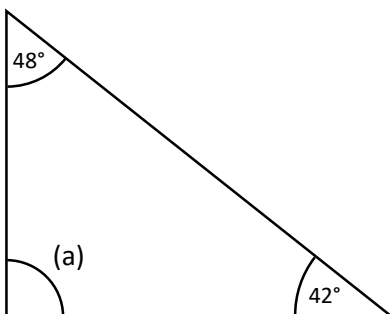
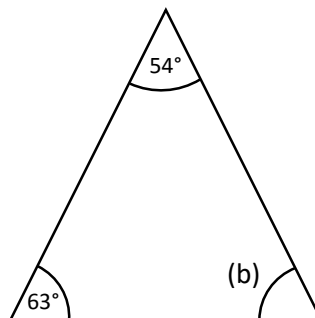
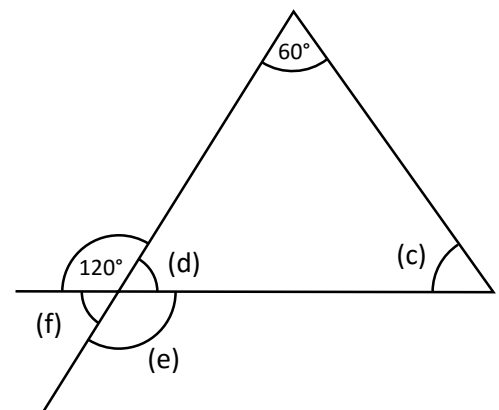
Use a ruler, a protractor and a compass to draw a triangle ABC of the following forms.

- (a) $AB = 5\text{cm}$; $\widehat{BAC} = 40^\circ$; $\widehat{ABC} = 60^\circ$.
 (b) $AB = 5.5\text{cm}$; $\widehat{BAC} = 30^\circ$; $\widehat{ABC} = 80^\circ$.
 (c) $AB = 6.2\text{cm}$; $\widehat{BAC} = 45^\circ$; $\widehat{ABC} = 36^\circ$.
 (d) $AB = 4.8\text{cm}$; $\widehat{BAC} = 27^\circ$; $\widehat{ACB} = 79^\circ$.
 (e) $AB = 6\text{cm}$; $BC = 8\text{cm}$; $\widehat{ABC} = 64^\circ$.
 (f) $AB = 5\text{cm}$; $BC = 6.2\text{cm}$; $\widehat{ABC} = 105^\circ$.
 (g) $AC = 7\text{cm}$; $BC = 6\text{cm}$; $\widehat{ACB} = 90^\circ$.
 (h) $AB = 4.7\text{cm}$; $BC = 4.7\text{cm}$; $AC = 4.7\text{cm}$.
 (i) $AB = 6.7\text{cm}$; $BC = 7.2\text{cm}$; $\widehat{ABC} = 38^\circ$.
 (j) $AB = 4.5\text{cm}$; $BC = 7.3\text{cm}$; $AC = 7.3\text{cm}$.
 (k) $AB = 5.7\text{cm}$; $BC = 8.1\text{cm}$; $AC = 9.2\text{cm}$.
 (l) $AB = 3.5\text{cm}$; $BC = 8\text{cm}$; $AC = 6\text{cm}$.



Exercise 26 (Revision)

Find the **size** of the marked angles. What **type** of triangles are shown? (The diagrams are not drawn to scale.)



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			<div> My Level <input type="text"/> My Target <input type="text"/> </div>